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Threshold of critical power in streams

WILLIAM B. BULL *Department of Geosciences, University of Arizona, Tucson, Arizona 85721*

ABSTRACT

Stream power is the power *available* to transport sediment load, and it may be defined as γQS, where γ is specific weight of water, Q is stream discharge, and S is slope. Critical power is the power *needed* to transport sediment load. The threshold of critical power is where stream power/critical power = 1.0. Where stream power exceeds critical power during long time spans, additional sediment load is obtained by vertical erosion that cuts V-shaped cross-valley profiles in bedrock. The threshold is approached asymptotically during downcutting, and high-order streams approach the threshold more rapidly than do low-order streams. High discharges cause net lateral erosion in reaches near the threshold. Straths and flood plains form under such conditions. Where stream power is less than critical power, selective bedload sedimentation decreases sediment load and size and therefore the critical power. Such deposition is self-enhancing because of concurrent decreases in slope. Thus, it is unlikely that aggrading reaches attain the threshold, but the tendency to attain the threshold may keep stream and critical power roughly the same. Reaches of streams at the critical-power threshold are sensitive to changes in climate, base level, and the impact of humans; these may change stream and/or critical power and result in aggradation or degradation.

INTRODUCTION

Although a model may be selected by a geomorphologist with the intent of making an efficient study and obtaining reasonable results, the background and biases of the investigator are important in determining the selection of a problem and the approach used. Gilbert (1877, 1914) chose to emphasize geomorphic processes. Davis (1899, 1902) chose to emphasize landform morphologies. Schumm and Lichty (1965) pointed out that time and space considerations (1) influence one's viewpoint regarding attainment of equilibrium in geomorphic systems, and (2) vary greatly between a study that emphasizes interaction of variables along a reach of a stream during several hours and a study that emphasizes morphologic changes of a drainage basin during millions of years.

Streams develop morphologies that depend on the frequency and magnitude of discharge of sediment and water from the hillslope subsystem. Some workers have regarded this interaction between form and process as an approximate equilibrium between the variables of the stream subsystem (Gugliemini, 1867; Surell, 1841; Dausse, 1872 — all cited in Rouse and Ince, 1957, p. 71; Davis, 1902; Mackin, 1948; Rubey, 1952; Hack, 1960. Other workers have emphasized the tendency toward adjustment between interdependent variables (Gilbert, 1914; Kesseli, 1941; Leopold and Maddock, 1953; Bull, 1975).

Most workers consider the concept of the graded stream as an equilibrium situation where, over a period of years, the hillslope subsystem supplies a uniform discharge of water and sediment to the stream subsystem. Because of the absence of long-term trends in discharge characteristics, the alluvial channel that has achieved a graded condition has developed a morphology so that the stream velocity is sufficient to transport the imposed sediment load (Mackin, 1948). Davis (1902) believed that grade was achieved only after a long time and that it was attained only in the mature and late stages of his "cycle of erosion." Knox (1976), who is interested in climatic change and humans as causes for ungraded streams, defined a graded stream as "one in which the relationship between process and form is stationary and the morphology of the stream remains constant over time." In contrast to the viewpoint of Davis, Knox believes that adjustment to a graded condition occurs rapidly. Leopold and Bull (unpub.) prefer to emphasize more than slope and velocity by defining equilibrium conditions in terms of stream power. They have stated that "a graded stream is one in which, over a period of years, slope, velocity, depth, width, roughness, pattern and channel morphology delicately and mutually adjust to provide the power and efficiency necessary to transport the load supplied from the drainage basin without net aggradation or degradation of the channels." This definition also includes the concept of how a graded stream differs from one that is not graded.

Although a tendency toward equilibrium conditions exists in streams, the attainment of graded conditions for long periods of time may be unlikely for many reaches of streams. Changes in independent variables of the fluvial system, such as climate, total relief as affected by tectonic movements, the erodibility of the surficial materials, and the human impacts create conditions conducive to change instead of equilibrium in fluvial systems. The time needed for changes in the above variables to affect the operation of the hillslope subsystem ranges from 10^6 yr for the effects of tectonic uplift in arid fluvial systems to 10 yr for the impact of humans where vegetation is cleared from hills in humid regions. Long time lags of response and adjustment for hillslope subsystems result in long time spans for stream subsystems to approximate graded conditions. Most fluvial systems now are responding to several changes in independent variables, each with its own time lag needed to approach a new equilibrium condition. Other landforms — such as deposits and topographic inversion — do not even tend toward equilibrium configurations (Bull, 1976a).

This study focuses on geomorphic thresholds, rather than on the concept of equilibrium, to explain the interrelations between process and form in fluvial systems. A geomorphic threshold is a transition point or period of time that separates different modes of operation within part of a landscape system. Adjustments within fluvial systems are further complicated by feedback mechanisms that interact with thresholds and produce complex responses within the system to perturbations (changes

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in independent variables). The interrelation between a threshold and feedback mechanism is outlined in Figure 1, A. Change in base level affects the gradient, and thereby stream power; which, in part, determines whether only sediment transport, or net aggradation or degradation, occurs at the foot of a hill.

The differences between the threshold and equilibrium concepts are illustrated in Figure 1, B. Geomorphic equilibrium occurs when self-regulating feedback mechanisms cause an adjustment among the variables of a system, or part of a system, such that changes in landscape morphology do not occur with time. Points in time that separate reversals in modes of operation are thresholds, but they are not equilibrium conditions unless an adjustment to a time-independent landform assemblage has occurred. Periods of equilibrium are thresholds when they separate different modes of operation of the system.

PURPOSE AND SCOPE

An important threshold — the threshold of critical power — separates the modes of net deposition and net erosion in fluvial systems. My purpose here is to analyze the critical-power threshold and to demonstrate the widespread application of the threshold approach to the understanding of the interrelations between processes and landforms.

First, the components of the threshold are analyzed, then the types of landscapes associated with downcutting and nondowncutting modes of operation of stream systems are outlined. Variations of stream systems in time and space as affected by the threshold are demonstrated by three markedly different examples. First, the responses of a fluvial system to tectonic uplift of a mountain front (a local perturbation) which

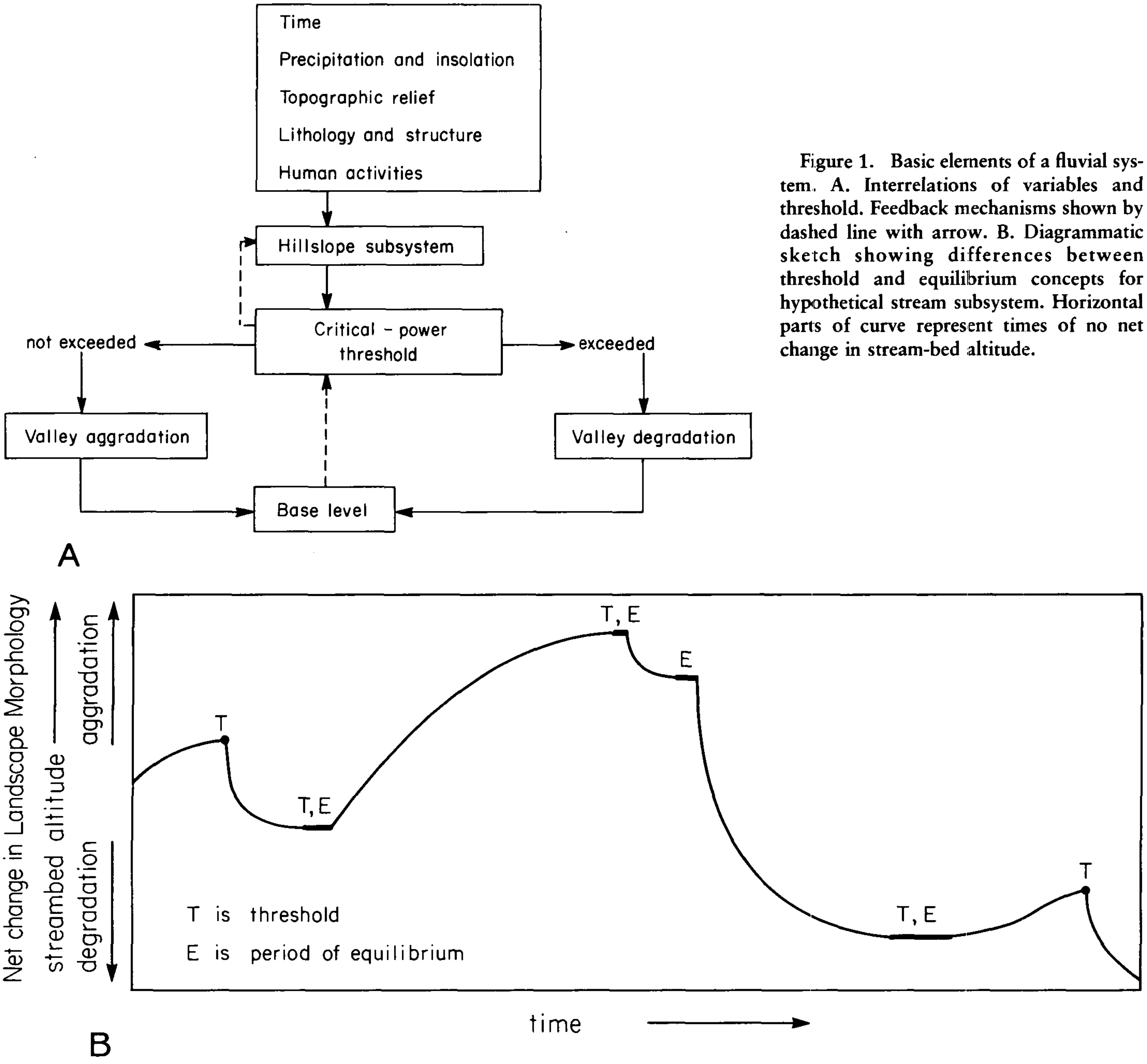


Figure 1. Basic elements of a fluvial system. A. Interrelations of variables and threshold. Feedback mechanisms shown by dashed line with arrow. B. Diagrammatic sketch showing differences between threshold and equilibrium concepts for hypothetical stream subsystem. Horizontal parts of curve represent times of no net change in stream-bed altitude.

involve geologic time spans (10^5 to 10^6 yr) are outlined. Second, the Pleistocene-Holocene climatic change that affects entire drainage basins but for shorter time spans (10^4 yr) are analyzed. Third, the impact of humans is considered, in the context of arroyo cutting, involving small spaces and time intervals (10 to 10^2 yr).

THRESHOLD OF CRITICAL POWER

Useful threshold concepts include those that stress adjustment to changing variables. The critical-power threshold separates the modes of erosion and deposition in streams and is dependent on the relative magnitudes of power needed to transport the average sediment load and on the stream power available to transport the load.

Streams may be regarded as sediment-transporting machines and may be analyzed in terms of the availability of stream power to do work (Bagnold, 1973, 1977; Emmett and Leopold, 1977). Stream power is dissipated in maintaining fluid flow against flow resistance and in doing work by moving the saltating bedload. Where stream power is more than sufficient to transport an imposed sediment load, scour of alluvium on the streambed, and perhaps of bedrock, may occur. Where stream power is insufficient, part of the saltating load will stop and the bed of the stream will aggrade. Bagnold described the kinetic power along a stream channel as γQS, where γ is the absolute density mass per volume, Q is discharge, and S is the gravity gradient. γ is assumed to be roughly constant, although it is recognized that sediment concentrations are decreased by ground-water additions to perennial streams and are increased by infiltration of ephemeral streams. It is useful to consider the total power supply per unit area of streambed, ω where

$$\omega = \gamma QS/\text{width} = \gamma dSu = \tau u, \qquad (1)$$

where d is the mean flow depth, u is the mean flow velocity, and τ is the mean boundary shear stress.

Stream power as defined by Bagnold places an emphasis on the availability of power to transport bedload. Definitions of power that emphasize flow velocity and slope (Yang, 1971; Stall and Yang, 1972) may be useful for analyses of meanders and pools and riffles but are not as useful as the Bagnold equation for the analysis of an erosion-deposition threshold.

The stream power *available* to transport sediment is one component of the critical-power threshold and consists of those variables that if increased favor transportation of the sediment. Stream power was selected as one component of the threshold because sediment transport is highly sensitive to changes in discharge and slope of water (for example, see Baker, 1973, Fig. 54).

The importance of discharge on stream power is dramatically revealed by the marked increases in sediment concentration that occur with increasing discharge at a station. Suspended sediment transport rates (G) may increase by the large exponential factor of about 2.5 with increase in discharge (Q) (Leopold and others, 1964, p. 220–221):

$$G = p\,Q^{2.5}. \qquad (2)$$

The other component of the threshold is critical power. Critical power is the stream power *needed* to transport the average sediment load supplied to a reach of a stream and consists of those variables that if increased favor deposition of the sediment. Critical power changes with variations in sediment load and size and with hydraulic roughness. The term "critical power" is a shorthand expression (through the continuity equation, $Q = wdv$) for variables such as width, depth, and velocity that affect hydraulic roughness and channel morphology. All of these variables interact to determine the capacity and competence of the stream to transport sediment.

Both stream and critical power change with time. Changes in stream power during short time spans generally are the result of changes in discharge. Rates of change of slope tend to be more conservative. Relatively rapid changes in slope occur with the changes in sinuosity that result from changes in stream-channel pattern. Downcutting or backfilling changes slope at a slower rate. Critical power may change rapidly with the amount and size of sediment load derived from the hillslope subsystem and with changes in hydraulic roughness. Changes in streamflow characteristics such as the ratio of water depth to sediment size (Bagnold, 1973, 1977) also affect the amount of power needed to transport bedload, but this type of change is in order to achieve maximum efficiency as a stream tends toward a graded (equilibrium) condition.

The threshold of critical power is defined as

$$\frac{\text{stream power}}{\text{critical power}} = 1.0 \qquad (3)$$

The components of the threshold described by equation 3 differ in their ease of measurement. Stream power may be estimated by measurements of discharge and stream gradient. Energy grade lines parallel the longitudinal profiles of the water surfaces for reaches of small streams that are more than 100 m long (Leopold and others, 1964, p. 304; Baker, 1974). Critical power includes hydraulic roughness, and, like the useful concept of hydraulic roughness, it cannot be measured directly in the field. Despite this apparent drawback, the ratio definition of the threshold is substantially more versatile than erosion-deposition thresholds stated merely in terms of available channel slope.

A simple application of the critical-power threshold is shown in Figure 2, which depicts a stream that has been affected by the emplacement of a road berm and a culvert north of Tucson, Arizona. The culvert was installed slightly higher than the stream bed and constitutes a minor local base-level rise. Reach A of the stream has local scour and backfill but no net aggradation or degradation, and thus it may be regarded as approximating a threshold (graded) condition. Aggradation postdating culvert emplacement has occurred in reach B, where stream power has become insufficient to transport the sediment load, as a result of decrease in slope (which is due partly to ponding during peak discharges). Critical power also increased in reach B as the aggrading area became more vegetated, thereby increasing hydraulic roughness. Reach C is not in equilibrium because much of the bedload has been trapped upstream from the culvert. This reduction in critical power has resulted in active channel downcutting of reach C, despite the concurrent decreases in slope downstream from the plunge pool associated with the culvert. Thus, in a distance of less than 1 km, reaches of a stream may be found that are at, are less than, and exceed the threshold of critical power.

Substantial philosophical differences exist between the threshold and graded-stream conceptual frameworks. The graded-stream approach seems most applicable for large spaces and long time spans, but the threshold concept may be applied to problems that vary greatly in both time and space. Both approaches consider the interaction between process and form, but the threshold concept emphasizes the possibility of change in a fluvial system. Those using the threshold approach are more likely to be interested in when and where change occurs in fluvial systems and the reasons for change, rather than searching for approximations of equilibrium. The graded-stream approach generally encourages study of self-regulating feedback mechanisms, but the threshold approach generally encourages study of self-enhancing feedback mechanisms. The graded-stream approach assumes that after a perturbation a stream will return to an equilibrium longitudinal profile. The critical-power threshold approach encompasses the equilibrium concept, but it emphasizes how far removed a stream is from equilibrium and recognizes that the behavior of both the stream and hillslope subsystems are dependent in part on the extent of deviation from the critical-power threshold (that is, the graded condition).

The ratio of vertical to lateral cutting during floods in alluvial stream channels is determined largely by how close the stream is to the critical-power threshold. In most cases, stream-bed scour is followed by backfilling during the waning stages of a flow event. These short-term changes are chiefly the result of changes in discharge and load. Where changes in slope occur, they are only temporary, because permanent changes in slope in a reach approximating the threshold might change the stream power sufficiently to cross the threshold. In reaches where stream power exceeds critical power, vertical erosion predominates, but lateral erosion predominates where a stream is close to the threshold. Lateral erosion tends to be permanent, as indicated by the presence of straths and flood plains.

Perennial streams may scour or backfill their channels during large flows, but low flows are times of reworking of those stream-bed materials that can still be transported. Ephemeral streams characteristically aggrade their channels during low flows because streamflows infiltrate into the channel before reaching the mouth of a drainage basin. Major flows may cause net scour of the channels of ephemeral streams as the accumulated sediment is flushed out of a given drainage net.

Long-term variations (> 1,000 yr) in critical power are the result of changes in amount and size of sediment discharge from the hillslope subsystem. Such changes most commonly are the result of climatic or base-level changes, although the impact of humans is important in many parts of the world.

For either long or short time spans, the interrelations of materials, processes, and landforms can be evaluated by using the allometric-change approach in which landscape elements are viewed as changing at different rates (Bull, 1975). The allometric approach allows for either graded or changing conditions. The critical-power threshold is defined allometrically in equation 3 because the relative power of the two components determines the threshold. Defining thresholds by using the format of equation 3 is advantageous (Bull, 1979). The components of the threshold are identified and compared to each other. The numerical index defines the relative conditions that must be met in order to cross the threshold and change the mode of system operation.

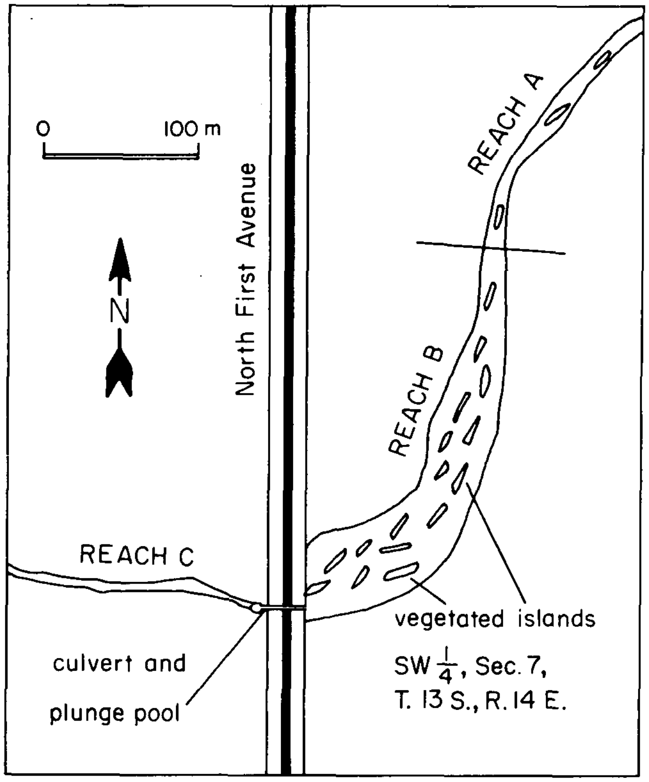


Figure 2. Sketch map showing variations in width of active channel of stream that has been affected by emplacement of road embankment and culvert; north of Tucson, Arizona.

PROCESSES AND MORPHOLOGIES

Three possible interrelations between stream and critical power are shown in Figure 3. The figure is not to scale and may be regarded either as variations that characteristically occur with stream order or as a common situation along trunk stream channels.

The hypothetical situation depicted in Figure 3 portrays the effects of local thunderstorm rainfall of 20 mm in 30 min falling on barren granitic hillslopes in the headwaters of a large drainage basin in an arid region. Stream power decreases with increasing distance from the headwaters. Maximum values of discharge and slope occur in reach A, but overall slope decreases downstream, and discharge decreases downstream as flow infiltrates into the dry steam bed. Discharge, and stream power, decrease to zero in reach C. Critical power increases in reach A as sediment load is picked up from the hillslopes and stream beds, decreases in reach B because of decreases in hydraulic roughness, and decreases in reach C because of decreases in load.

Changes in power for ephemeral and perennial streams can be compared by using the average exponents of the downstream hydraulic geometry equations (Leopold and others, 1964, p. 244). For the ephemeral stream system depicted in Figure 3, $w \alpha Q^{-0.5}$, $d \alpha Q^{-0.3}$, $u \alpha Q^{-0.2}$, and $S \alpha Q^{-0.8}$.

Total stream power, Ω, decreases markedly:

$$\Omega \alpha wduS, \tag{4}$$

$$\Omega \alpha Q^{(-0.5-0.3-0.2-0.8)},$$

$$\Omega \alpha Q^{-1.8}, \tag{5}$$

and stream power per unit width, ω, also decreases:

$$\omega \alpha duS \tag{6}$$

$$\omega \alpha Q^{-1.3} \tag{7}$$

Discharge increases downstream in perennial streams, and $w \alpha Q^{+0.5}$, $d \alpha Q^{+0.4}$, $u \alpha Q^{+0.1}$, and $S \alpha Q^{-0.8}$.

Total stream power increases,

$$\Omega \propto Q^{(+0.5+0.4+0.1-0.8)}$$

$$\Omega \propto Q^{+0.2},$$

but stream power per unit width decreases:

$$\omega \propto Q^{-0.3}. \tag{9}$$

Headwaters streams in most mountainous regions generally exceed the critical-power threshold. Stream power is much more than is needed to transport the sediment load and overcome roughness in reach A of Figure 3, part A. Cross-valley morphologies of such reaches characteristically are V-shaped because the stream obtains additional sediment by vertical erosion into bedrock. All downcutting reaches, however, have a tendency to approach the threshold of critical power.

Stream power is less than the critical power in reach C of Figure 3 — a ratio of less than 1. Both steep and gentle reaches may occur in locally aggrading reaches. In Figure 4, deposition of sand has occurred in a locally aggrading section of a bedrock channel. In reach X the mode of operation is to alluviate the channel and valley floor and represents the situation depicted in reach C of Figure 3. Increases in flow width, infiltration capacity, and vegetation all act as self-enhancing feedbacks that promote additional alluviation. Stream power in reach X does not tend to remain less than critical power, because selective sedimentation decreases sediment load and size, thereby reducing the critical power and tending to re-establish the critical-power threshold. In order to achieve the threshold, the decrease in sediment load must be sufficient to compensate for the concurrent decrease in stream gradient caused by alluviation. It is unlikely that aggrading reaches attain the threshold, but the tendency to attain the threshold may result in roughly similar values of stream and critical power.

The deposition of the patch of alluvium illustrated in Figure 4 also results in the formation of reach Y, which is inherently unstable because the stream slope is steep. Channel entrenchment into the alluvium may occur, particularly at high discharges. The formation of channels tends to concentrate flow, and this is a self-enhancing feedback that tends to destroy the patch of alluvium.

Thus, local aggradation may result in reaches that either exceed or are less than the critical-power threshold and where the relative rates of change of processes and landforms are dependent on two offsetting self-enhancing feedback mechanisms. Alluviation will be temporary in a bedrock channel such as illustrated in Figure 4, and where streams debouch onto a permanent depositional area, such local alluviations are redistributed over the surface of the deposit.

Stream power and critical power are equal, but changing, in reach B, which is in equilibrium. Stream power is decreasing because of decreases in discharge and slope. Sediment load is constant or may even increase, but, by definition, it cannot decrease until reach C. Hydraulic adjustments act as self-regulating feedback mechanisms to maintain graded conditions in reach B despite decreases in discharge. If dune bedforms and highly turbulent flow are present in reach A, they may give way to the planar beds of reach B. The resulting decreases in hydraulic roughness cause decreases in critical power and provide an example where

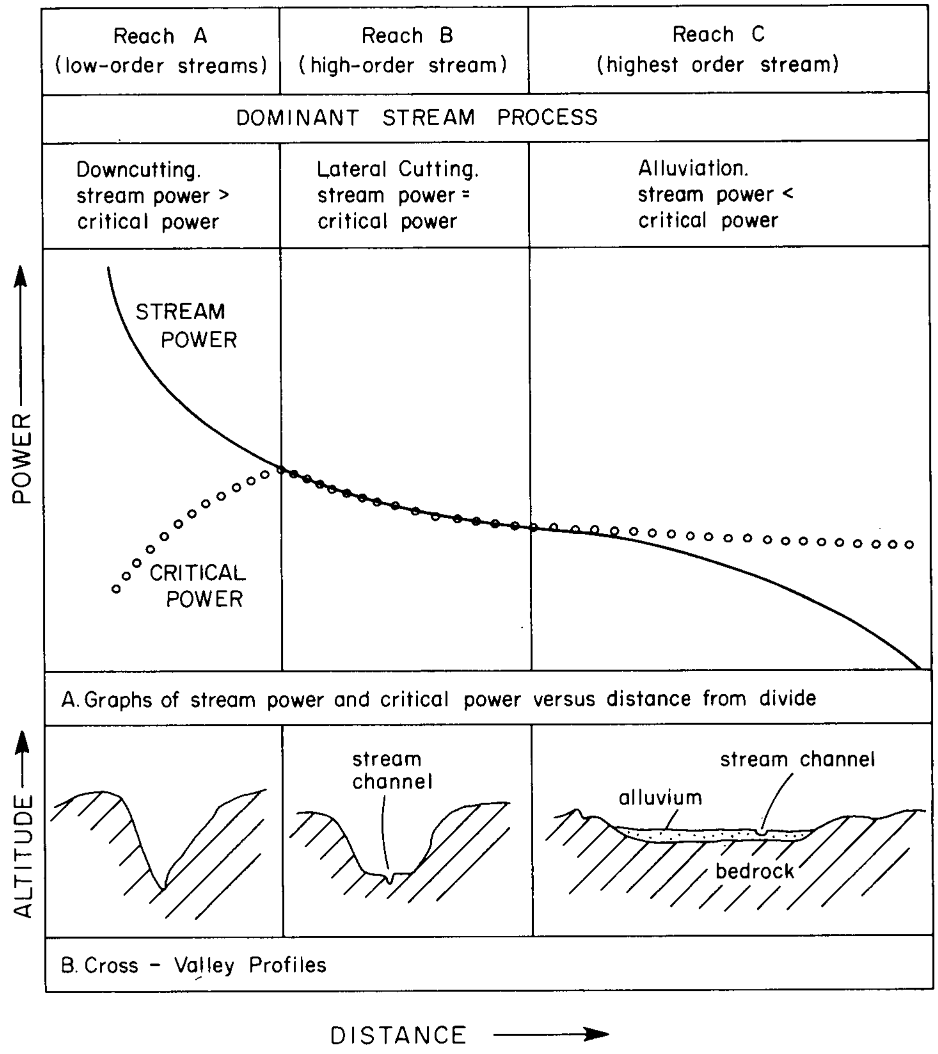


Figure 3. Diagrammatic sketches and graphs of stream power and critical power for arid rocky drainage basin.

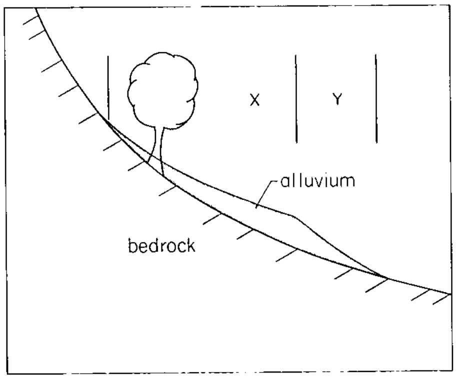


Figure 4. Diagrammatic sketch of stream profile showing adjacent alluvial reaches that are more gentle (X) and steeper (Y) than pre-existing bedrock channel.

hydraulic adjustments are sufficient to maintain a graded-stream condition despite concurrent changes in several variables. In reaches A and C of Figure 3, part A, the hydraulic adjustments are insufficient to allow attainment of graded streamflow.

A reach where stream power is more than the critical power will tend to erode down to the threshold of critical power. High-order streams achieve the threshold condition more rapidly than do low-order streams because of their greater capacity. The rate of downcutting decreases asymptotically, and lateral erosion and deposition become more important as the threshold is approached. Minor downcutting or deposition may occur in a reach, but such local processes are temporary and commonly are offset by the presence of the opposite process within the same reach, as in a point-bar environment. Straths form under such conditions. This concept was first stated by Gilbert (1877, p. 126): "Downward wear ceases when the load equals the capacity for transportation. Whenever the load reduces the downward corrasion to little or nothing, lateral corrasion becomes relatively and actually of importance."

A variety of field evidence may indicate that a given reach of a stream is close to the critical-power threshold. The presence of alluvium in amounts that exceed that scoured by large discharges suggests that net vertical erosion is minimal. In downcutting reaches, stream width at peak discharges equals valley-floor width; but when lateral cutting becomes predominant over downcutting, the floodplain is narrower than the valley-floor width. Measurements that show neither net erosion nor deposition indicate threshold conditions. For time spans of 1 to 100 yr, measurements of erosion and deposition can be made in the field. For longer time spans, radiogenic dating of stratigraphy may be used. The threshold has been passed if accelerated downcutting occurs as a result of minor steepening of the channel due to base-level fall or local alluviation. Parallel stream terraces may be suggestive of a return to similar threshold conditions after adjustments to perturbations. The evidence that many depositional settings were close to the threshold is found in stratigraphies that contain numerous temporary small hiatuses.

Reaches of streams at the critical-power threshold are highly susceptible to accelerated downcutting or alluviation because of changes in either stream or critical power. For the situation depicted in reach B of Figure 3, part B, a moderate increase in the critical power may result in alluviation. A moderate decrease in critical power may accelerate the rate of channel downcutting. Changes in the critical power that result from changes in the independent variables are a major cause of passing the threshold, which results in alluviation or terracing of streams. The situation is different for reach A, where even a large increase in critical power can occur and the stream will continue to downcut. For reach C, changes in critical power may (1) accelerate the rate of alluviation, (2) return the mode of operation to equilibrium conditions, or (3) cause the threshold to be crossed, thereby initiating entrenchment of the channel into the alluvium.

The concept that streams tend toward the minimum gradients needed to transport their sediment loads has been recognized by many workers (such as Leopold and Langbein, 1962; Yang, 1971, p. 243) and is an important part of the graded stream and critical-power threshold conceptual frameworks. A graded stream would be one that has attained and remained at the critical-power threshold. Knox (1976) would consider a stream to be graded even if long-term net erosion or deposition were taking place. Knox's approach pertains to those streams that remain on one side or the other of the critical-power threshold or those that remain at the threshold.

VARIATIONS IN TIME AND SPACE

In this section the critical-power threshold is used to evaluate time lags in arid fluvial systems that have responded to perturbations of greatly different character and duration. The topics include the assessment of the impact of tectonic uplift, climatic change, and human actions.

Responses to Tectonic Perturbations

Differential vertical uplift at a mountain front is a perturbation that first affects the fluvial system adjacent to the front. Headcut migration steepens the drainage net and then the hillslopes. The ridge crests in the headwaters of the drainage basin will be the last landscape element to adjust to the increase in relief caused by the uplift.

Uplift rates of mountain fronts are not uniform. Periods of rapid uplift are separated by periods of minimal tectonism, when stream erosion is the chief local base-level process. After substantial uplift, the critical-power threshold may be exceeded along an entire drainage net, indicated by lack of net alluviation in narrow V-shaped valleys. Straths formed during periods of tectonic quiescence will become terraces with the onset of the next period of accelerated uplift, which steepens the slope of the active stream channel.

An example of a stream that has repeatedly returned to the critical-power threshold after pulses of differential uplift of the mountain front is the Wadi Saada, which discharges onto a large alluvial fan along the coast of the east-central Sinai Peninsula. The differential uplift appears to be chiefly downfaulting of the rift valley to the east. Most of the drainage basin is underlain by coarse-grained granitic rocks, and hydrolytic weathering and salt splitting

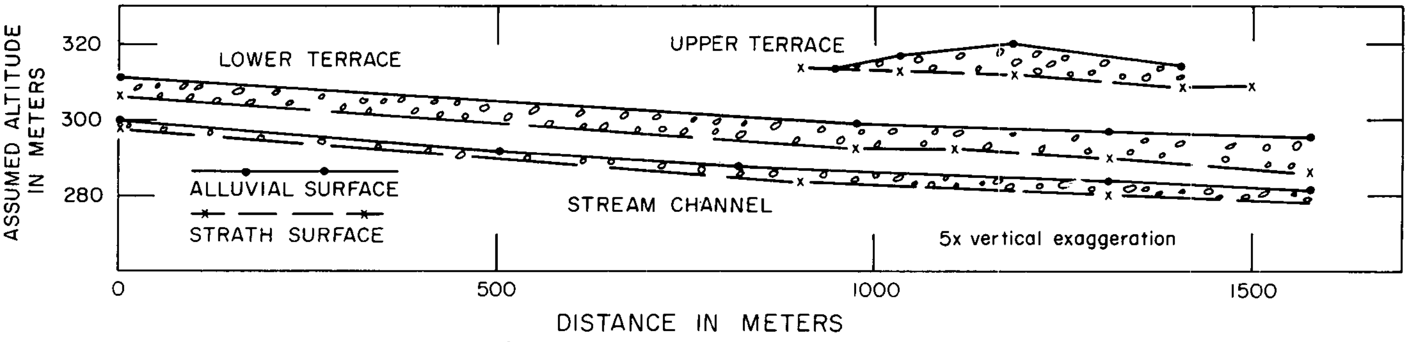


Figure 5. Longitudinal profiles of strath terraces of Wadi Saada, east-central Sinai Peninsula.

are important processes that produce large amounts of grus.

Although the width of the Wadi Saada exceeds 100 m at the mountain front, it is a strath that is overlain by 3 m of bouldery gravel. Figure 5 shows remnants of two similar ancestral straths now preserved under terrace gravels at about 10 and 30 m above the wadi strath surface.

Although a net increase in the total relief of the watershed has occurred as a result of the uplifts, permanent steepening of the stream gradient apparently has not occurred in the reach immediately upstream from the fault scarp. Differential uplift of the mountain front caused headward erosion in the reach upstream from the fault and establishment of a steeper and a narrower valley than before faulting. Then, during a period of tectonic inactivity, fan deposition constituted a base-level rise in the reach downstream from the fault. The stream cut down to the critical power threshold upstream from the mountain front, and lateral erosion widened the valley floor. The presence of three straths suggests that long periods of tectonic quiescence occurred between uplifts of 20 and 10 m. The mean discharge of water and sediment from the hillslopes may not have changed much during the long time spans represented by the suite of terraces. The similarity of terrace slopes may reflect similar sizes of the coarse-grained bedload (Leopold and Bull, unpub.).

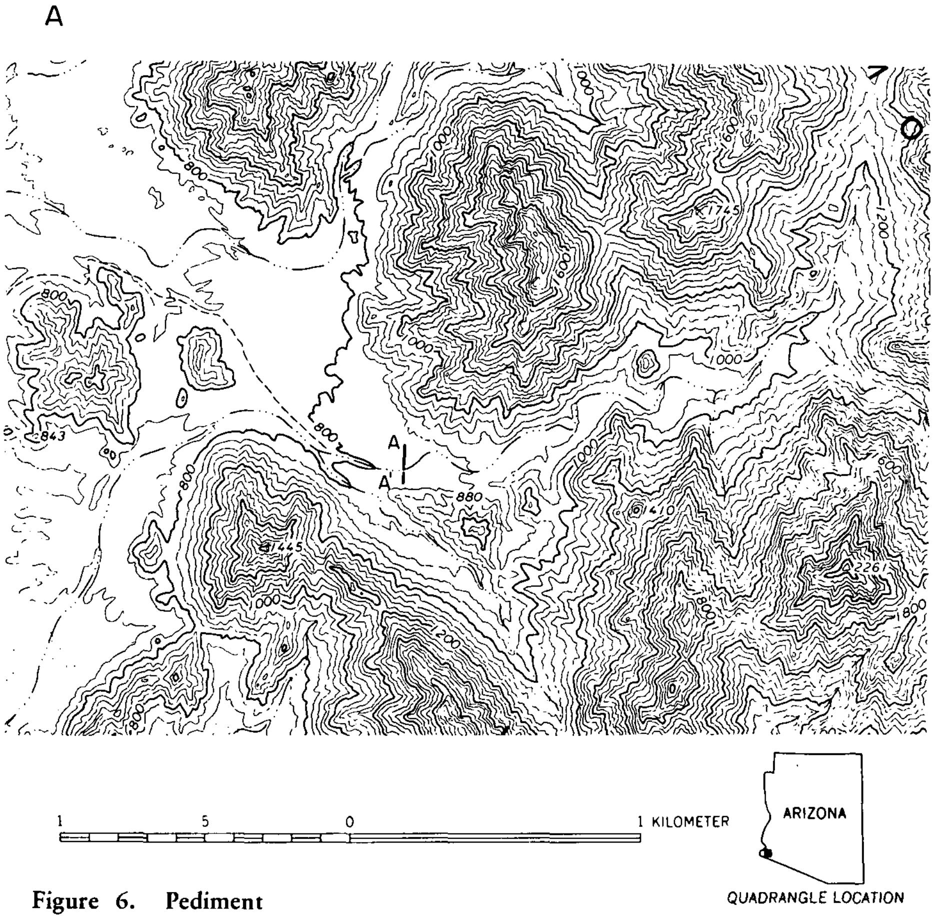
The rates of headcut migration — a type of accelerated vertical erosion — will determine the rate of upstream migration of the effects of a tectonic perturbation. The mountain-front reach is the first to approach the threshold of critical power, and the progressive increase of the ratio of lateral to vertical erosion results in valley widening near the mountain front while active downcutting is still occurring upstream.

The rate of valley-floor narrowing with distance upstream from a mountain front can be expressed by the power function

$$W = c\,L^n, \qquad (10)$$

where L is distance upstream from the mountain front, and W is width of the valley floor. Scatter about the regressions (Fig. 6, B) is largely the result of variations in erosional widening of the valley floors caused by nonuniform lithology and structure and changes in valley width where tributary streams join the trunk stream. The coefficient, c, is indicative of the valley-floor width at 100 m upstream from the start of the transect, which is shown by line A–A′ in Figure 6, A. The exponent, n, is indicative of the rate of valley-floor narrowing.

During the valley downcutting that occurs after mountain-front uplift, the width of the valley floor will approximate stream width at high discharges. Valley-floor width decreases upstream from the front because of the decrease in the size of the contributing watershed. During the initial downcutting of the valley, c will be an index of the magnitude of peak stream discharges at a unit distance (100 m in this case) upstream from the mountain front. With the passage of geologic time, the stream will widen its valley as it approaches the threshold of critical power. As lateral cutting becomes progressively more important, the stream will not spread over the entire valley floor during high discharges. The approximation of a threshold condition migrates gradually upstream as the upstream reaches downcut, so that the stream and critical power are roughly the same for time spans of 10^4 yr. The configurations of the plan views of the valley mouths — the pediment embayments of Figure 6, A — are functions of the rates



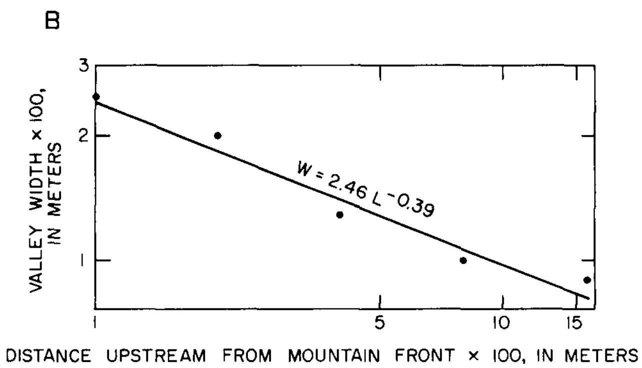


Figure 6. Pediment embayments of Gila Mountains, Arizona. A. Topographic map from Fortuna Mine quadrangle. Data for part B were collected from embayment whose mouth is marked by A–A′. Rocks are mafic gneiss, pegmatite, and quartz diorite. B. Graph showing narrowing of valley width (W) with increasing distance (L) upstream from mountain front.

of lateral cutting and/or hillslope retreat along the stream and the time elapsed since lateral erosion became predominant at the various points along the valley. More than a million years may be needed to form pediment embayments.

The values of the exponent of equation 10 commonly range from −0.1 to −1.0, but most of the exponents range from −0.1 to −0.4. These low rates of decrease of valley-floor width relative to distance upstream from the mountain front suggest that (1) the rates of migration of attainment of the threshold condition upstream from the fronts commonly have been moderately rapid (the streams' tectonically steepened gradients decreased fairly rapidly during and after cessation of uplift), and/or (2) the low rate of narrowing is controlled by structures that parallel the valley. This is not surprising, because many streams owe their locations partly to the greater ease of erosion along zones of abundant joints and shears.

Such long periods of time are needed for entire drainage nets to achieve the threshold condition that it may not happen. The adjustment time is longer for upstream reaches than reaches at the mountain front because of decreasing stream capacity in the upstream direction. In Figure 6, part A, the headwaters streams have yet to cut down to the threshold condition. Pediment-embayment development is an example of an extremely long time lag in response to progressive decrease in the stream-power component of the threshold. However, the reaches of the stream that are close to or on the erosional side of the threshold can be identified easily.

Responses to a Climatic Perturbation

In the section on pediment embayment, I discussed changes in space of the critical-power threshold during time spans of 10^6 yr, as affected by a perturbation in only a small part of the system — the zone of differential uplift at the mountain front. This section emphasizes variation of threshold conditions during 10^4 where the perturbation of Pleistocene-Holocene climatic change occurred throughout arid fluvial systems.

The change to Holocene climates in the hot deserts of the Middle East and the American Southwest can be generalized by stating that precipitation decreased and/or temperature increased. These changes in the independent variables caused the following postulated sequence of changes in the arid fluvial systems. Both climatic changes reduced the moisture available for plant growth. Reduction of vegetative density decreased infiltration rates and exposed more soil to erosion, resulting in increases of sediment concentration and runoff of water (Fig. 7) for a precipitation event of a given amount and intensity. Increases in sediment load and size greatly increased the critical power. The increase in critical power was

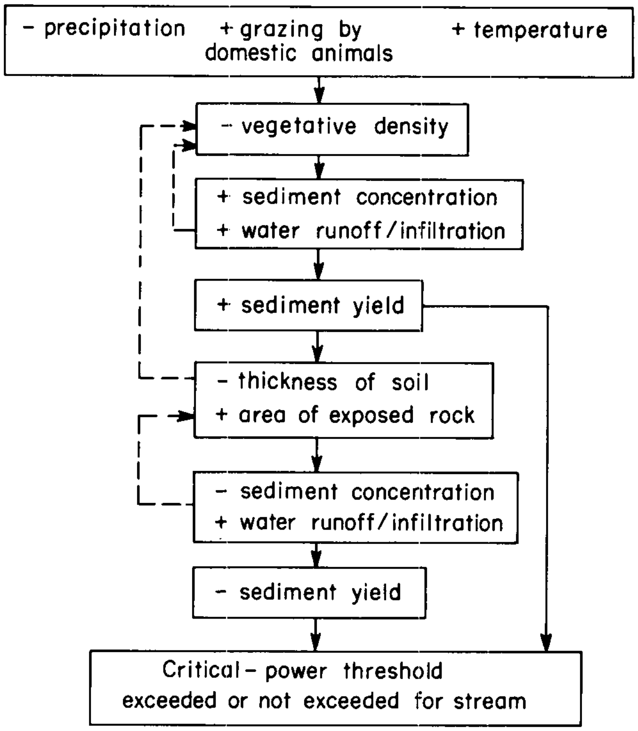


Figure 7. Increases (+) and decreases (−) in elements of hypothetical arid hillslope subsystem. Self-enhancing feedback mechanisms are shown by dashed line with arrow.

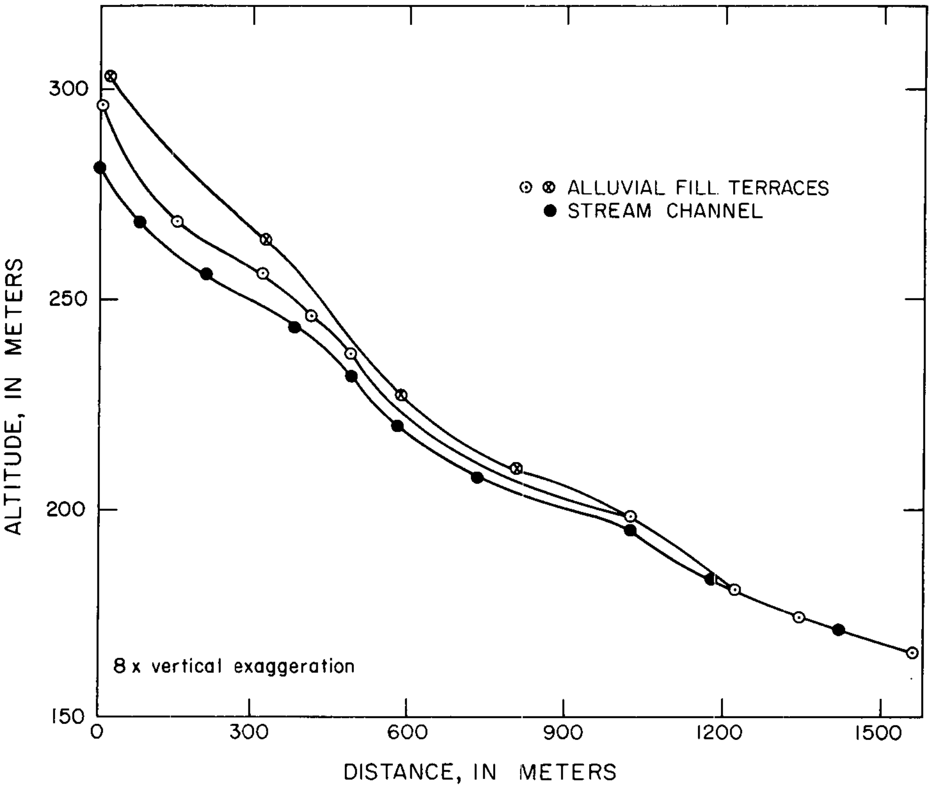


Figure 8. Longitudinal profiles of converging fill terraces in Riverside Mountains, California.

sufficient to maintain a condition where the critical-power threshold was not exceeded, despite increases in some stream gradients caused by valley alluviation. Decrease in soil thickness and concurrent increase in area of exposed bedrock caused still more rapid runoff of water, and the sediment concentration decreased as more bare rock was exposed (Fig. 7). The resulting decrease in sediment yield reduced the critical power, but the stream power had been increased by the deposition of the steep valley fill. The critical-power threshold was crossed as a result of the changes occurring in the hillslope subsystem, and erosion of the valley fill began. Three self-enhancing feedback mechanisms tended to perpetuate the net removal of soil from the slopes (Fig. 7). Increased flashiness of runoff continued to decrease soil thickness, which resulted in continued decrease in vegetative density.

Fill terraces, such as those of Figure 8, occur in the arid parts of the Mojave and Sonoran Deserts of Arizona and California. Valley fills were 6 to 30 m thick, but nearly all the streams now are downcutting into bedrock. The widespread occurrence of three Holocene terrace levels reflects climatic variations during Holocene time, but these have been minor compared to the Pleistocene-Holocene climatic change. Plants collected and stored by pack rats (*Neotoma* sp.) provided Van Devender (1973, 1977) abundant fossils from plant communities, and materials to date the times of climatic change. For western Arizona he concluded that starting about 8,000 yr ago annual precipitation decreased about 50%, that most of the decrease occurred during the winter rainy season, and that the mean annual temperature increased about 3 °C.

The fluvial systems have been changing as a result of the climatic change. The single major perturbation resulted in consecutive valley alluviation and downcutting as self-enhancing feedback mechanisms changed stream and critical power. The stream subsystem changed modes of operation in a classic example of what Schumm (1973) has referred to as complex response of fluvial systems. Holocene alluviation temporarily increased stream gradients in the Mojave Desert by as much as 25%. Although the changes in stream discharge and gradient caused large changes in stream power, the changes in critical power resulting from changes in sediment load and size were even larger and occurred more rapidly.

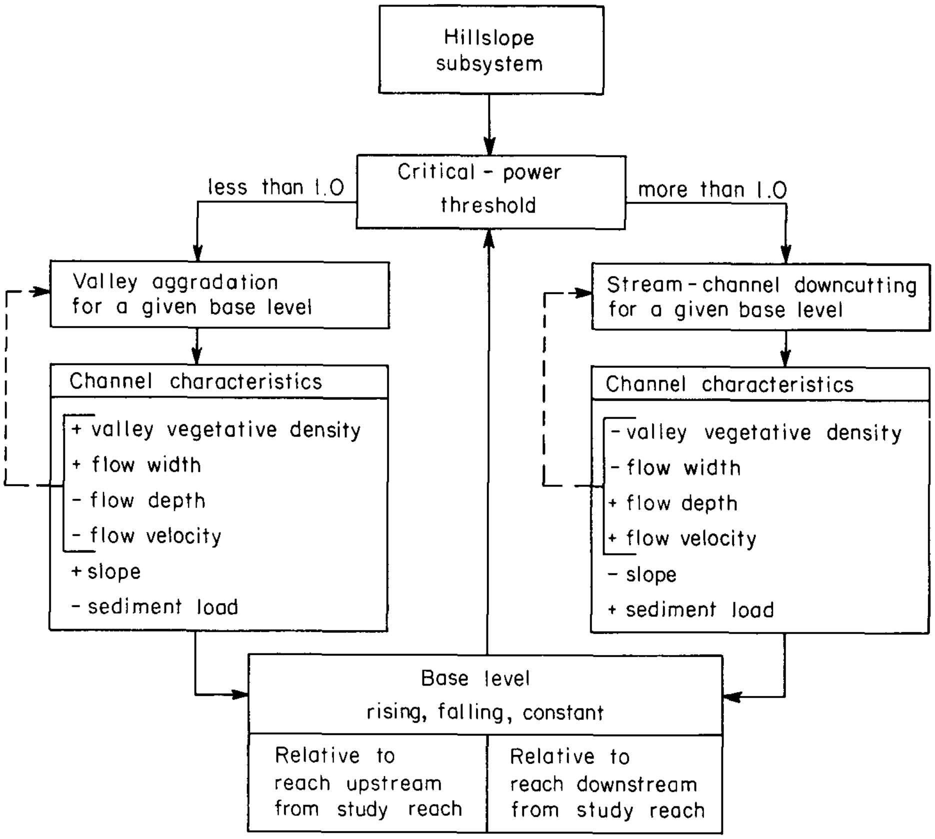


Figure 9. Increases (+) and decreases (−) in elements of hypothetical semiarid stream subsystem. Self-enhancing feedback mechanisms are shown by dashed line with arrow.

Response to Impact of Humans

The response to grazing — or other impacts such as short climatic variations — is most pronounced in semiarid stream systems underlain by fine-grained, easily eroded materials. Changes in sediment load and hydraulic roughness are large and commonly occur during time spans of 10 to 100 yr.

The critical-power threshold separates the two modes of operation of such stream systems (Fig. 9). Where the threshold is exceeded for a stream such as reach Y of Figure 4, decreases in valley vegetative density and flow width, and increases in flow depth and velocity all tend to act as self-enhancing feedbacks to perpetuate the downcutting mode. Increase in sediment load and decrease in slope tend to offset the effects caused by changes in the above four variables. Most entrenching streams downcut rapidly, approximate threshold conditions for a while, and then backfill or renew downcutting in response to new changes in the independent variables or to complex responses (Schumm, 1973) of the system. The valley aggradation mode (Fig. 9) has changes in dependent variables that are opposite those of the downcutting mode. For either mode, changes in base level directly affect the critical-power threshold.

An example of the sensitivity of such streams to the impact of humans is provided by the Dead Mesquite Wash study area (Packard, 1974) near Tucson, Arizona. A discontinuous ephemeral stream supports a lush growth of trees, bushes, and grass where streamflow spreads out on channel fans that are sites of valley aggradation. Self-enhancing feedbacks promote vegetative growth where vegetation greatly spreads and reduces velocity of streamflow, thereby causing deposition of additional clayey soil and prolonged infiltration of streamflow. Grazing, fire, or encroachment by headcuts in the adjacent downstream reach cause the critical-power threshold to be exceeded and establish an opposite self-enhancing feedback mechanism. The change is particularly pronounced in clay-rich soils because the initiation of any minor channel greatly decreases residence time of ephemeral sheet flow and, thereby, the infiltration of water to support the vegetation. Within decades lush growth is transformed into badlands studded with bleached tree trunks (Fig. 10).

Patton and Schumm (1975) studied discontinuous gullies in the Piceance Basin of northwestern Colorado, where they found sandstone, siltstone, and marlstone to be the most common hillslope rock types. They compared slopes and drainage areas (a proxy for discharge) of gullied and ungullied reaches (Fig. 11), and their work showed that channel entrenchment occurred when, for a given drainage area, alluviation steepened the reach of a stream above a threshold slope in much the same manner as for reach Y of Figure 4.

In Figure 11, Patton and Schumm's plot has been divided into three groups of points in order to demonstrate the relative importance of the two components of the critical-power threshold: stream power and the critical power. The solid line is an approximation of the critical-power threshold for the different stream reaches of the Piceance Basin. Variations in stream power dominate the threshold for the points in areas A and B. None of the reaches of area B has sufficiently steep gradients that the threshold is exceeded, and virtually all of the reaches in area A are entrenched. Critical power does not vary much for the reaches of areas A and B, thereby allowing a clear relation between valley slope, discharge (basin area), and the presence of entrenched or unentrenched streams.

The relation between valley slope and basin area does not hold for area C, which consists of steep reaches with source areas of less than 20 km^2. There is a good reason for the critical power to vary more in the reaches of area C, and therefore be a more important determinant of whether or not entrenchment has occurred in the reaches of area C. The denser hillslope and valley-floor vegetation of those small basins dominated by north-facing slopes (Patton and Schumm, 1975, p. 89) has increased the hydraulic roughness and decreased discharge so that critical power is larger than stream power. Thus, the mode of operation of some, but not all, of the small basins has been alluviation instead of entrenchment of the valley floors.

Figure 11 is useful for analysis of potential impact of humans on their environment. The critical-power threshold is identified for a study area, and the relative importances of critical and stream power for different reaches of the fluvial system can be determined. Individual reaches such as point S are identified; they appear to be especially sensitive to increases in the

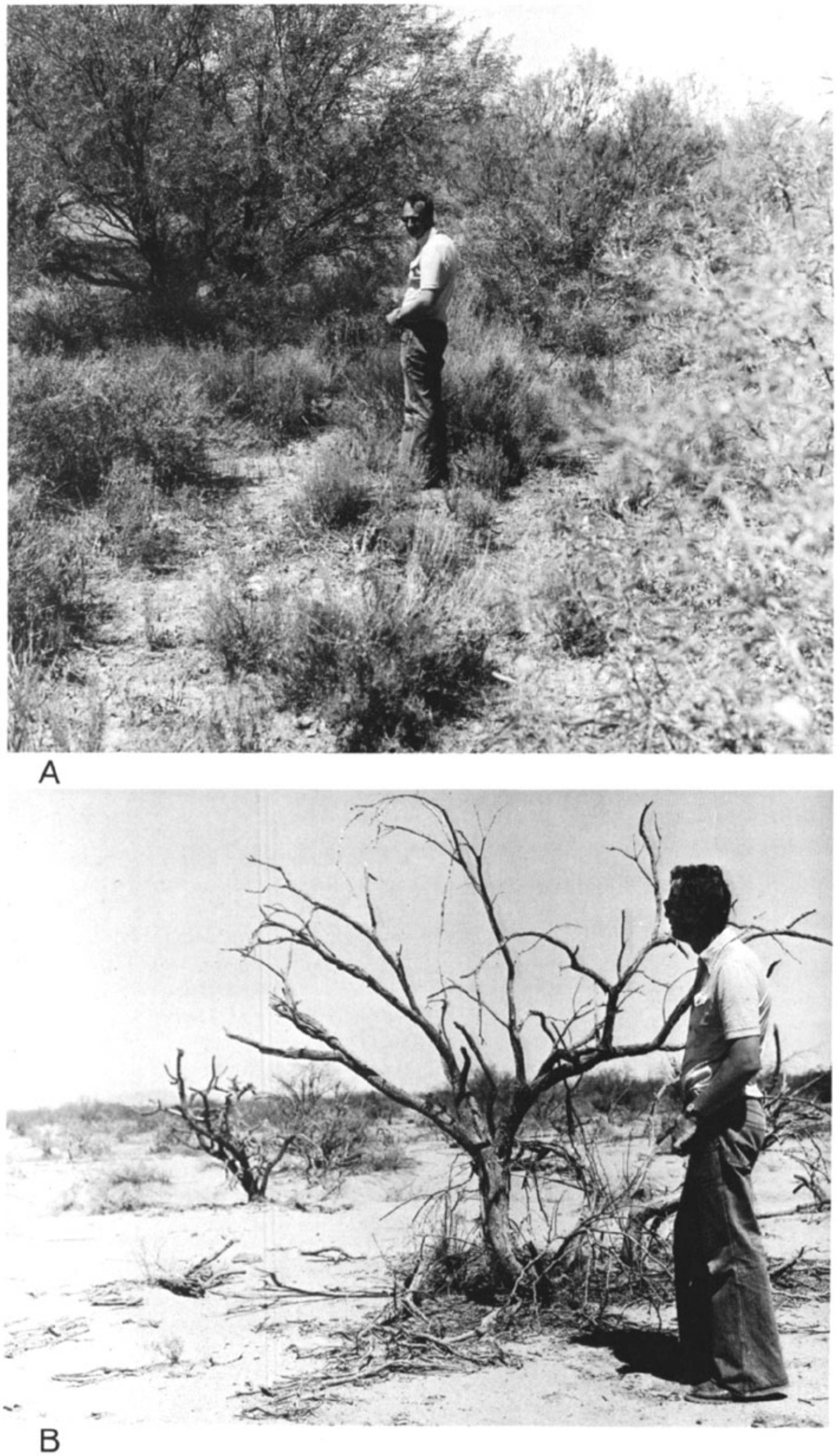


Figure 10. Threshold relations for discontinuous ephemeral stream. Dead Mesquite Wash study site, Arizona. A. Densely vegetated reach at critical-power threshold. B. Barren reach adjacent to reach shown in A. Critical-power threshold has been exceeded for this reach.

stream power or decreases in critical power and, thereby, are likely to be entrenched.

CONCLUSIONS

The concept that streams tend toward uniform and minimum expenditure of power needed to transport their sediment loads (Leopold and Langbein, 1964) is an important part of the conceptual frameworks that emphasize equilibrium (the graded stream) or change in fluvial systems (the threshold of critical power). Some of the differences in emphasis of the two approaches are as follows. (1) Thresholds can be used in studies involving investigations that range from minutes to millions of years and for spaces of equally great contrast, but the graded-stream concept applies primarily to long times and large spaces. (2) The threshold approach tends to focus attention on those variables and complex responses that are likely to cause the mode of system operation to change. (3) The threshold approach generally encourages study of self-enhancing feedback mechanisms, whereas the graded-stream approach generally encourages study of self-regulating feedback mechanisms.

Identification of thresholds in studies of fluvial systems promotes versatility of approach and emphasis of those variables that are likely to cause the mode of system operation to change. The use of thresholds encourages study of the relative rates of change of variables — allometric change — and de-emphasizes consideration of situations that may be unlikely, such as the attainment of equilibrium (steady state) for long periods of time.

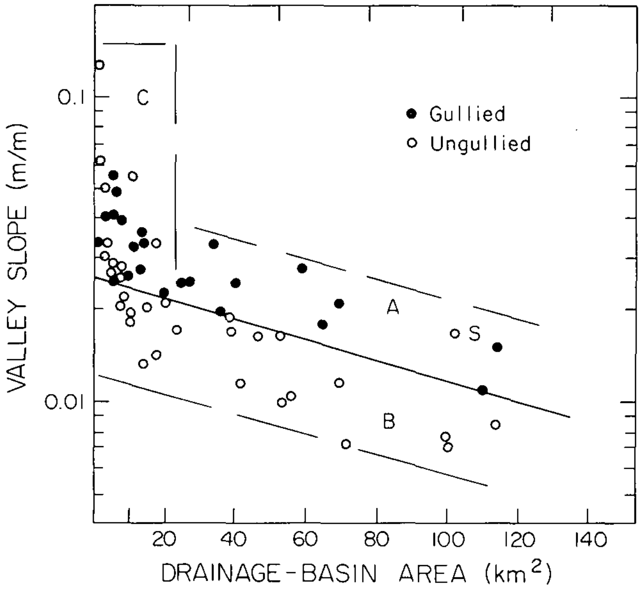


Figure 11. Relation of valley slope to drainage-basin area for gullied and ungullied reaches of discontinuous ephemeral stream in Piceance Creek Basin, Colorado (modified from Patton and Schumm, 1975, Fig. 2). Solid line is critical-power threshold and separates reaches that have exceeded threshold (A) from reaches that have yet to exceed threshold (B). Small watersheds of area C have mixed characteristics due to greater variability of critical power. Point S is ungullied reach that is considered to be especially susceptible to channel entrenchment.

Two conditions relating to the critical-power threshold can be recognized easily in the field: (1) reaches where the threshold has been exceeded, and (2) reaches that approximate the threshold or where the critical power exceeds the stream power. Active downcutting by the stream and lack of evidence for alluviation are clear evidence that the threshold is being exceeded. The following field situations indicate that a stream is close to the critical-power threshold: (1) the presence of alluvium in amounts that exceed that scoured by large discharges, (2) a floodplain that is narrower than the valley-floor width, (3) measurements of dated alluvial sequences that indicate neither net erosion nor deposition for a reach, (4) parallel fill, or strath, terraces, which suggest fluctuating conditions and periodic return to similar threshold conditions, and (5) numerous, small hiatuses of a temporary nature in the stratigraphy of a valley fill.

It is desirable to use ratios when defining thresholds. The critical-power threshold is where stream power/critical power = 1.0. Such a ratio is an allometric approach, because the relative importances of two aspects of the system are used to define the threshold. The ratio format clearly defines the relative conditions needed to change the mode of system operation.

The critical-power threshold occupies a key position in the complex interactions between the hillslope and stream subsystems, and it is affected by feedback mechanisms and complex responses operating in either subsystem. Recognition of how far removed a stream is from the critical-power threshold should aid in better understanding of both landscape morphologies and processes, as well as their interrelations.

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REFERENCES CITED

Bagnold, R. A., 1973, The nature of saltation and of bed-load transport in water: Royal Society of London Proceedings, ser. A, p. 473–504.

——1977, Bed load transport by natural rivers: Water Resources Research, v. 13, p. 303–312.

Baker, V. R., 1973, Paleohydrology and sedimentology of Lake Missoula flooding in eastern Washington: Geological Society of America Special Paper 144, 79 p.

——1974, Paleohydraulic interpretation of Quaternary alluvium near Golden, Colorado: Quaternary Research, v. 4, p. 94–112.

Bull, W. B., 1975, Allometric change of landforms: Geological Society of America Bulletin, v. 86, p. 1489–1498.

——1976a, Landforms that do not tend toward steady state, *in* Flemal, R., and Melhorn, W., eds., Theories of landform development: Binghamton, N.Y., Annual Geomorphology Symposia Series, 6th, Proceedings, p. 111–128.

——1976b, The critical-gradient threshold: Geological Society of America Abstracts with Programs, v. 8, p. 795.

——1979, Geomorphic thresholds as defined by ratios, *in* Vitek, J., and others, eds., Thresholds in geomorphology: Stroudsburg, Pa., Dowden, Hutchinson & Ross, Inc. (in press.)

Davis, W. M., 1899, The geographical cycle: Geographical Journal, v. 14, p. 481–504.

——1902, Base-level, grade, and peneplain: Journal of Geology v. 10, p. 77–111.

Emmett, W. W., and Leopold, L. B., 1977, A comparison of observed sediment-transport rates with rates computed using existing formulas, *in* Doehring, D. O., ed., Geomorphology in arid regions: Binghamton, N.Y., Annual Geomorphology Symposia Series, 8th, Proceedings, p. 187–188.

Gilbert, G. K., 1877, Report on the geology of the Henry Mountains: U.S. Geographical and Geological Survey of the Rocky Mountain Region, 160 p.

——1914, The transportation of debris by running water: U.S. Geological Survey Professional Paper 86, 259 p.

Hack, J. T., 1960, Interpretation of erosional topography in humid temperate regions: American Journal of Science, v. 258-A, p. 89–97.

Kesseli, J. E., 1941, The concept of the graded river: Journal of Geology, v. 49, p. 561–588.

Knox, J. C., 1976, Concept of the graded stream, *in* Flemal, R., and Melhorn, W., eds., Theories of landform development: Binghamton, N.Y., Annual Geomorphology Symposia Series, 6th, Proceedings, p. 169–198.

Leopold, L. B., and Langbein, W. B., 1962, The concept of entropy in landscape evolution: U.S. Geological Survey Professional Paper 500-A, 20 p.

Leopold, L. B., and Maddock, Thomas, 1953, The hydraulic geometry of stream channels and some physiographic implications: U.S. Geological Survey Professional Paper 252, 57 p.

Leopold, L. B., Wolman, M. G., and Miller, J. P., 1964, Fluvial processes in geomorphology: San Francisco, W. H. Freeman, 522 p.

Mackin, J. H., 1948, Concept of the graded river: Geological Society of America Bulletin, v. 59, p. 463–512.

Packard, F. A., 1974, The hydraulic geometry of a discontinuous ephemeral stream on a bajada near Tucson, Arizona [Ph.D. dissert.]: Tucson, University of Arizona, 127 p.

Patton, P. C., and Schumm, S. A., 1975, Gully erosion, northwestern Colorado: A threshold problem: Geology, v. 3, p. 88–90.

Rouse, H., and Ince, S., 1957, History of hydraulics: Iowa City, Iowa Institute of Hydraulic Research, 269 p.

Rubey, W. W., 1952, Geology and mineral resources of the Hardin and Brussels quadrangles (in Illinois): U.S. Geological Survey Professional Paper 218, 175 p.

Schumm, S. A., 1973, Geomorphic thresholds and complex response of drainage systems, *in* Morisawa, M., ed., Fluvial geomorphology: Binghamton, N.Y., Annual Geomorphology Symposia Series, 4th, Proceedings, p. 229–310.

Schumm, S. A., and Lichty, R. W., 1965, Time, space, and causality in geomorphology: American Journal of Science, v. 263, p. 110–119.

Stall, J. B., and Yang, C. T., 1972, Hydraulic geometry and low streamflow regimen: Illinois State Water Survey Research Report no. 54, 31 p.

Van Devender, T. R., 1973, Late Pleistocene plants and animals of the Sonoran Desert: A survey of ancient packrat middens in southwestern Arizona [Ph.D. dissert.]: Tucson, University of Arizona, 179 p.

——1977, Holocene woodlands in the southwestern deserts: Science, v. 198, p. 189–192.

Yang, C. T., 1971, On river meanders: Journal of Hydrology, v. 13, p. 231–253.

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